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06CS42

Fourth Semester B.E. Degree Examination, June/July 2011

Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions selecting at least TWO questions from each part.***

PART – A

1 a. Define complete bipartite graph. How many vertices and how many edges are there $K_{4,7}$ and $K_{7,11}$? **(05 Marks)**

b. If a graph with n vertices and m edges is k-regular, show that m = kn/2. Does there exist a cubic graph with 15 vertices. **(05 Marks)**

c. Verify that the two graphs shown below in Fig.Q1(c)(i) and Fig.Q1(c)(ii) are isomorphic.

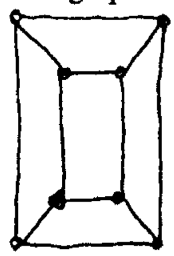


Fig.Q1(c)(i)

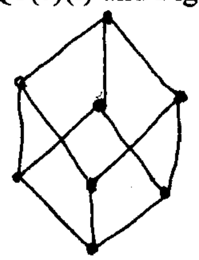


Fig.Q1(c)(ii) **(05 Marks)**

d. If G is a simple graph with no cycles, prove that G has atleast one pendant vertex. **(05 Marks)**

2 a. Prove that Petersen graph is non-planar. **(04 Marks)**

b. Prove that a connected planar graph G with n vertices and m edges has exactly m – n + 2 regions in every one of its diagrams. **(06 Marks)**

c. Show that every simple connected planar graph G with less than 12 vertices must have a vertex of degree ≤ 4. **(05 Marks)**

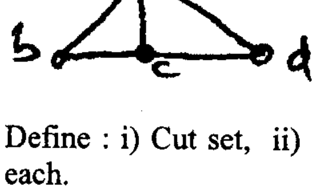
d. Prove that every connected simple planar graph G is 6 colourable. **(05 Marks)**

3 a. Prove that a tree with n vertices has n – 1 edges. **(07 Marks)**

b. Obtain a prefix code for the message 'ROAD IS GOOD', using lebelled binary tree and hence encode the message. **(07 Marks)**

c. Define a spanning tree of a graph. Find all the spanning trees of the following graph shown in Fig.Q3(c). **(06 Marks)**

Fig.Q3(c)



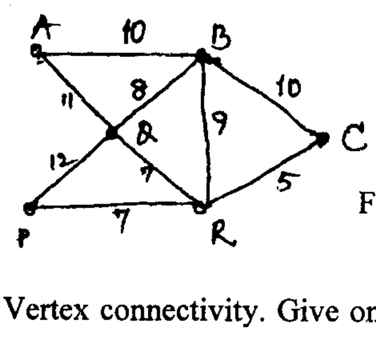


Fig.Q4(b)

4 a. Define : i) Cut set, ii) Edge connectivity, iii) Vertex connectivity. Give one example for each. **(06 Marks)**

b. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown in Fig.Q4(b). **(07 Marks)**

c. State and prove max-flow and min-cut theorem. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. In how many ways one can distribute ten identical white marbles among six distinct containers? **(06 Marks)**

b. Prove the following identities :

i) $C(n+1, r) = C(n, r-1) + C(n, r)$

ii) $C(m+n, 2) - C(m, 2) - C(n, 2) = mn$. **(07 Marks)**

c. Determine the coefficient of :

i) xyz^2 in the expansion of $(2x - y - z)^4$

ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$ **(07 Marks)**

6 a. There are 30 students in a hostel. In that 15 study history, 8 study economics, and 6 study geography. It is known that 3 students study all these subjects Show that 7 or more students study none of these subjects. **(06 Marks)**

b. In how many ways can one arrange the letters in CORRESPONDENTS so that:

i) There is no pair of consecutive identical letters.

ii) There are exactly two pairs of consecutive identical letters.

iii) There are atleast three pairs of consecutive identical letters? **(08 Marks)**

c. Define derangement. In how many ways we can arrange the numbers 1, 2, 3, ...10 so that 1 is not in the 1^{st} place, 2 is not in the 2^{nd} place and so on, and 10 is not in the 10^{th} place? **(06 Marks)**

7 a. Determine the generating function for the numeric function :

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$ **(06 Marks)**

b. Find the coefficient of x^{18} in the following products :

$$(x + x^3 + x^5 + x^7 + x^9)(x^3 + 2x^4 + 3x^5 + ...)^3$$ **(07 Marks)**

c. In how many ways can we distribute 24 pencils to 4 children so that each child gets at least 3 pencils but not more than eight? **(07 Marks)**

8 a. Solve the recurrence relation, $F_{n+2} = F_{n+1} + F_n$, given $F_0 = 0$ and $F_1 = 1$ and $n \geq 0$. **(06 Marks)**

b. Find the generating function for the relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$, with $a_0 = -1$ and $a_1 = 8$. **(07 Marks)**

c. Find the general solution of $s(k) + 3s(k-1) - 4s(k-2) = 4^k$. **(07 Marks)**

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